PROPAGATION OF THERMOELASTIC ACCELERATION WAVES IN MATERIALS WITH THERMAL MEMORY

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A method of surfaces of discontinuity is used to obtain expressions for velocities and attenuations of thermoelastic waves propagating in a semibounded medium with thermal memory.

A wide variety of thermophysical and mechanical properties of materials presently synthesized require for their description the usage of new representations that take into account the prehistory of the materials. The same is also required for describing the behavior of ordinary materials under extreme conditions, for example, materials undergoing intensive high-speed processes and materials at low temperatures. Therefore, a thermodynamic theory of materials accounting for thermal memory has attracted increasing attention recently as an effective method for describing a wide class of real media.

In this work, the wave modes of propagation of thermoelastic perturbations are considered in the framework of a linearized coupled theory of thermoelasticity for isotropic media with allowance for thermal memory [1]. The determining relations for the thermal flux q_z, internal energy e, and stresses σ_{zz} are of the form

$$q_z(z,\ t)=q=\int\limits_0^\infty \alpha'(s)\overline{g_z^t}(z,\ s)\,ds, \tag{1}$$

$$e(z,\ t)=e=e_0+c_v\vartheta(z,\ t)-\int\limits_0^\infty \beta'(s)\overline{\vartheta^t}(z,\ s)\,ds+\varkappa_1 u_{,z}(z,\ t), \tag{2}$$

$$\sigma_{zz}(z,\ t)=\sigma=(2\varkappa_3+\varkappa_4)u_{,z}(z,\ t)-\varkappa_2\vartheta(z,\ t)-\int\limits_0^\infty \gamma'(s)\overline{\vartheta^t}(z,\ s)\,ds, \tag{3}$$

where $\vartheta = (T - T_0)/T_0 \ll 1$; $\varepsilon = u_{,z} \ll 1$.

Relaxation functions $\alpha(t)$, $\beta(t)$, and $\gamma(t)$, defined on the interval $t \in [0, \infty)$, are differentiable functions equal to zero at infinity.

The complete histories of the temperature $\overline{\vartheta^t}$ and the temperature gradient $\overline{g^t}$ are defined as:

$$\overline{f^t}(z,\ s)=\int\limits_0^s f^t(z,\ s)=ds,\ \ s\in[0,\ \infty), \tag{4}$$

$$f^t(z,\ s)=f(z,\ t-s),\ \ s\in[0,\ \infty). \tag{5}$$

Below, for an analysis of the propagation of thermoelastic waves we use an approach in which a wave is considered as a surface of discontinuity of thermodynamic quantities.

Assume that at the moment of time t, the position of the surface of discontinuity is determined by the coordinate $z = Y(t)$ with the velocity of the motion of the surface being equal to $u_n = dY(t)/dt$. The discontinuity of the function f(z, t) is designated by square brackets and is defined as:

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$$[f(t)] = f^-(t) - f^+(t) = \lim_{z \to Y^-(t)} f(z,\ t) - \lim_{z \to Y^+(t)} f(z,\ t).$$

Kinematic conditions of compatibility for one-dimensional plane waves according to the Maxwell theorem are written in the form

$$\frac{d}{dt}[f] = \left[\frac{\partial f}{\partial t}\right] + u_n\left[\frac{\partial f}{\partial z}\right]. \tag{6}$$

For weak waves, when the derivatives of the thermodynamic functions have discontinuities, while the functions themselves are continuous, from Eq. (6) we obtain

$$\left[\frac{\partial f}{\partial t}\right] = -u_n\left[\frac{\partial f}{\partial z}\right]. \tag{6'}$$

By definition, the surface of discontinuity Σ is called an acceleration wave if the field of the displacements u and the temperature T possess the following properties on it:

I) u is smooth and ϑ is a continuous function in the neighborhood μ of the surface Σ;

II) $\ddot{u}$, $\dot{\varepsilon}$, $\varepsilon_{,z}$, $\dot{\vartheta}$, and g are continuous in the region $\Sigma-\mu$ and undergo discontinuities when crossing Σ.

From I and II and conditions (6) it follows that

$$u_n[\dot{u}_{,z}] = -[\ddot{u}],\ u_n[\vartheta_{,z}] = -[\dot{\vartheta}]. \tag{7}$$

From (1)-(3) and properties I and II, it is seen that the fields q, e, and σ undergo discontinuities when crossing the front of the acceleration wave. It follows from the equations of balance of energy and momentum

$$\dot{e} = -q_{,z} + w, \tag{8}$$

$$\rho\ddot{u} = \sigma_{,z} + b, \tag{9}$$

that

$$[\dot{e}] = -[q_{,z}], \tag{10}$$

$$[\sigma_{,z}] = \rho[\ddot{u}]. \tag{11}$$

By applying the Maxwell theorem (6') to (10) and (11), we obtain

$$-[\dot{\sigma}] = \rho u_n[\ddot{u}], \tag{12}$$

$$u_n[\dot{e}] = [\dot{q}]. \tag{13}$$

From (1)-(3) and the property $\alpha(\infty) = \beta(\infty) = \gamma(\infty) = 0$, it follows that

$$\dot{q} = -\alpha(0)\,g(z,\ t) - \int_0^\infty \alpha'(s)\,g^t(z,\ s)\,ds, \tag{14}$$

$$\dot{e} = c_v\dot{\vartheta}(z,\ t) + \beta(0)\,\vartheta(z,\ t) + \int_0^\infty \beta'(s)\,\vartheta^t(z,\ s)\,ds + \varkappa_1\dot{u}_{,z}, \tag{15}$$

$$\dot{\sigma} = (2\varkappa_3 + \varkappa_4)\,\dot{u}_{,z} - \varkappa_2\dot{\vartheta} + \gamma(0)\,\vartheta - \int_0^\infty \gamma'(s)\,\vartheta^t(z,\ s)\,ds. \tag{16}$$

Then, according to Eqs. (14)-(16) and properties I and II, it follows that

$$[\dot{q}] = -\alpha(0)\,[g], \tag{17}$$

$$[\dot{e}] = c_v[\dot{\vartheta}] + \varkappa_1[\dot{u}_{,z}], \tag{18}$$

$$[\dot{\sigma}] = (2\varkappa_3 + \varkappa_4)\,[\dot{u}_{,z}] - \varkappa_2[\dot{\vartheta}]. \tag{19}$$

Substituting (17)-(19) into (12) and (13), we obtain the system of equations

$$\{u_n^2\rho - (2\varkappa_3 + \varkappa_4)\}[\ddot{u}] - \varkappa_2 u_n [\dot{\vartheta}] = 0, \tag{20}$$

$$\{u_n^2 c_v - \alpha(0)\}[\dot{\vartheta}] - \varkappa_1 u_n [\ddot{u}] = 0. \tag{21}$$

Thus, if the jumps in the derivative of temperature with respect to time $[\dot{\vartheta}] \neq 0$ and accelerations $[\ddot{u}] \neq 0$, then we obtain the equation for determining the velocities of propagation of two thermoelastic acceleration waves

$$\left(\frac{u_n}{c_1}\right)^4 - (b^2 + 1 + \varepsilon^*)\left(\frac{u_n}{c_1}\right)^2 + b^2 = 0, \tag{22}$$

where

$$\frac{u_n}{c_1} = \frac{1}{\sqrt{2}}\{1 + \varepsilon^* + b^2 \pm [(1 + \varepsilon^* + b^2)^2 - 4b^2]^{1/2}\}^{1/2}.$$

$$c_1^2 = (2\varkappa_3 + \varkappa_4)/\rho;\ c_2^2 = \alpha(0)/c_v;$$

$$b^2 = c_2^2/c_1^2;\ \varepsilon^* = \varkappa_1\varkappa_2/(2\varkappa_3 + \varkappa_4)c_v.$$

Let us determine the coefficients of attenuation of the thermoelastic waves. The jumps in the second derivatives of temperature and heat flow can be written in the form [2]

$$[\ddot{\vartheta}] = u_n^2[\vartheta_{,zz}] + 2\frac{d[\dot{\vartheta}]}{dt}, \tag{23}$$

$$[\dot{g}] = [\dot{\vartheta}_{,z}] = -u_n[\vartheta_{,zz}] - \frac{1}{u_n}\frac{d[\dot{\vartheta}]}{dt}, \tag{24}$$

$$u_n[\dot{q}_{,z}] = -[\ddot{q}] + \frac{d[\dot{q}]}{dt}. \tag{25}$$

Taking into account the relation (10), derived from the law of energy conservation, we rewrite (25) as

$$u_n[\ddot{e}] = -[\ddot{q}] + \frac{d}{dt}[\dot{q}]. \tag{26}$$

From (15) and (14), it follows that

$$\ddot{e} = c_v\ddot{\vartheta}(z,\ t) + \beta(0)\dot{\vartheta}(z,\ t) - \int_0^\infty \beta'(s)\frac{\partial}{\partial s}\vartheta^t(z,\ s)\,ds + \varkappa_1\ddot{u}_{,z}, \tag{27}$$

$$\ddot{q} = -\alpha(0)\dot{g}(z,\ t) + \int_0^\infty \frac{\partial}{\partial s}g^t(z,\ s)\,ds = -\alpha(0)\dot{g}(z,\ t) - \alpha'(0)g(z,\ t) - \int_0^\infty \alpha''(s)g^t(z,\ s)\,ds. \tag{28}$$

From (27) and (28), we obtain the jumps in the second derivatives $\ddot{q}$ and $\ddot{e}$

$$[\ddot{e}] = c_v[\ddot{\vartheta}] + \beta(0)[\dot{\vartheta}] + \varkappa_1[\ddot{u}_{,z}], \tag{29}$$

$$[\ddot{q}] = -\alpha(0)[\dot{g}] - \alpha'(0)[g]. \tag{30}$$

Substituting (29) and (30) into (28) and taking into account Eqs. (17), (23), and (24) and the Maxwell theorem (6), we find

$$2u_n c_v\frac{d[\dot{\vartheta}]}{dt} = \frac{\alpha'(0)}{u_n}[\dot{\vartheta}] - u_n\beta(0)[\dot{\vartheta}] - u_n\varkappa_1[\ddot{u}_{,z}] + \{\alpha(0)u_n - u_n^3 c_v\}[\vartheta_{,zz}]. \tag{31}$$

To eliminate $[\ddot{u}_{,z}]$ from (31), we use the relation

$$[\ddot{\sigma}] = u_n^2[\sigma_{,zz}] + 2\frac{d[\dot{\sigma}]}{dt}, \tag{32}$$

which we rewrite, making use of the law of conservation of momentum (9) and the Maxwell theorem (6):

$$[\ddot{\sigma}] = \rho u_n^2 [\ddot{u}_{,z}] + 2\frac{d[\dot{\sigma}]}{dt}. \tag{33}$$

We obtain the derivatives of stresses with respect to time from (3):

$$\dot{\sigma} = (2\varkappa_3 + \varkappa_4)\dot{u}_{,z} - \varkappa_2\dot{\vartheta} + \gamma(0)\vartheta + \int_0^\infty \gamma'(s)\vartheta^t(z, s)\,ds, \tag{34}$$

$$\ddot{\sigma} = (2\varkappa_3 + \varkappa_4)\ddot{u}_{,z} - \varkappa_2\ddot{\vartheta} + \gamma(0)\dot{\vartheta} + \int_0^\infty \gamma'(s)\frac{\partial}{\partial s}\vartheta^t(z, s)\,ds, \tag{35}$$

from which it follows that

$$[\dot{\sigma}] = (2\varkappa_3 + \varkappa_4)[\dot{u}_{,z}] - \varkappa_2[\dot{\vartheta}], \tag{36}$$

$$[\ddot{\sigma}] = (2\varkappa_3 + \varkappa_4)[\ddot{u}_{,z}] - \varkappa_2[\ddot{\vartheta}] + \gamma(0)[\dot{\vartheta}]. \tag{37}$$

Substituting (36) and (37) into (33), we obtain

$$\{(2\varkappa_3 + \varkappa_4) - u_n^2\rho\}[\ddot{u}_{,z}] = \varkappa_2[\ddot{\vartheta}] - \gamma(0)[\dot{\vartheta}] - 2\varkappa_2\frac{d[\dot{\vartheta}]}{dt} + 2(2\varkappa_3 + \varkappa_4)\frac{d}{dt}[\dot{u}_{,z}]. \tag{38}$$

From (9), (17), and (18), we have

$$[\dot{u}_{,z}] = \frac{\alpha(0)}{\varkappa_1 u_n^2}[\dot{\vartheta}] - \frac{c_v}{\varkappa_1}[\ddot{\vartheta}]. \tag{39}$$

Using (23), (36), and (39), we rewrite (31) as

$$2\left\{u_n c_v + (2\varkappa_3 + \varkappa_4)\frac{u_n}{(2\varkappa_3 + \varkappa_4) - u_n^2\rho}\left(\frac{\alpha(0)}{u_n^2} - c_v\right)\right\}\frac{d[\dot{\vartheta}]}{dt} +$$
$$+\left\{u_n\beta(0) - \frac{\alpha'(0)}{u_n} - \frac{u_n\varkappa_1\gamma(0)}{(2\varkappa_3 + \varkappa_4) - u_n^2\rho}\right\}[\dot{\vartheta}] + u_n\left\{\alpha(0) - u_n^2 c_v - \frac{u_n^2\varkappa_1\varkappa_2}{(2\varkappa_3 + \varkappa_4) - u_n^2\rho}\right\}[\vartheta_{,zz}] = 0. \tag{40}$$

It should be noted that, taking (22) into account, the expression in braces before $[\vartheta_{,zz}]$ is equal to zero. Therefore,

$$\frac{d[\dot{\vartheta}]}{dt} + \delta[\dot{\vartheta}] = 0, \tag{41}$$

where the coefficient of attenuation δ is of the form

$$\delta = \frac{u_n^2\beta(0) - \alpha'(0)}{2\alpha(0)}\,\frac{\left(\frac{u_n}{c_1}\right)^2 - 1}{\left(\frac{u_n}{c_1\sqrt{b}}\right)^4 - 1} + \frac{\varkappa_1\gamma(0)u_n^2}{2\alpha(0)(2\varkappa_3 + \varkappa_4)\left[\left(\frac{u_n}{c_1\sqrt{b}}\right)^4 - 1\right]}. \tag{42}$$

We write the coefficients of attenuation on the wave fronts in dimensionless form as

$$\bar{\delta}_{1,2} = \frac{\delta}{\left(\frac{u_n}{c_1}\right)}\left\{\frac{N\left(\frac{u_n}{c_1}\right)^2 - A}{2\left(\frac{u_n}{c_1}\right)}\,\frac{\left(\frac{u_n}{c_1}\right)^2 - 1}{\left(\frac{u_n}{c_1\sqrt{b}}\right)^4 - 1} + \Gamma\varepsilon^*\frac{\left(\frac{u_n}{c_1}\right)}{2\left[\left(\frac{u_n}{c_1\sqrt{b}}\right)^4 - 1\right]}\right\}. \tag{43}$$

$$N = \frac{\beta(0)}{\alpha(0)}a;\quad A = \frac{\alpha'(0)a}{\alpha(0)c_1^2};\quad \Gamma = \frac{\gamma(0)a}{\varkappa_2 c_1^2}.$$

Thus, in the framework of the model of linear thermoelasticity, taking account of thermal memory [1], the attenuating thermoelastic waves propagate with velocities (22) equal to the velocities obtained in the model of thermoelasticity, with account of the relaxation of the heat flow [3-5], where accounting for the additional mechanisms of relaxation of the internal energy and stresses leads to an increase in the coefficients of attenuation of the waves.

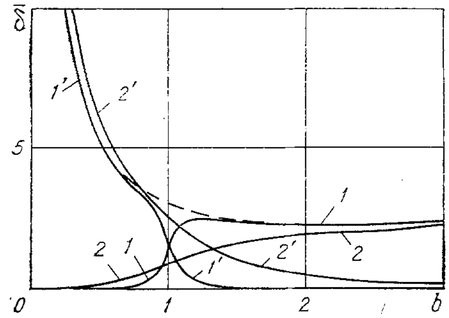

Fig. 1. Dependence of the attenuation coefficients $\bar{\delta}_{1,2}$ on the dimensionless velocity of propagation of thermal perturbations b: 1) $\bar{\delta}_1$; 1') $\bar{\delta}_2$ at ε = 0.0114; 2) $\bar{\delta}_1$; 2') $\bar{\delta}_2$ at ε = 0.432; N = 1, A = −5, Γ = 0.

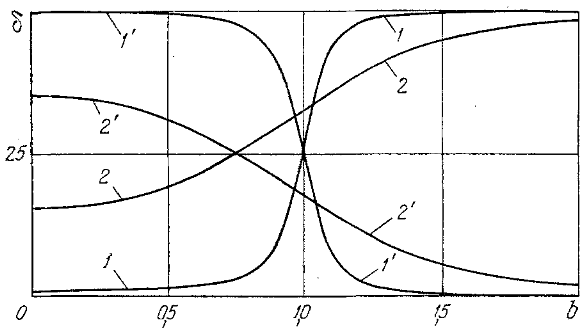

Fig. 2. Attenuation coefficients $\delta_{1,2}$ as a function of the dimensionless velocity of propagation of thermal perturbations b (the Maxwell functions of relaxation): 1) δ_1; 1') δ_2 for ε = 0.0114; 2) δ_1; 2') δ_2 for ε = 0.432.

Figure 1 shows the dependence of the coefficients of attenuation on the dimensionless velocity of propagation of thermal perturbations for materials with small (ε^* = 0.0114, steel) and large (ε^* = 0.432, polyvinylbutyral) coupling parameters (neglecting thermal relaxation of stresses Γ = 0). As is seen from the accompanying relationships, the slow wave is attenuated strongly for small velocities of heat propagation, while attenuation of the fast wave is close to zero. At high velocities of heat propagation, the fast wave is attenuated strongly, while attenuation of the slow wave is very small. In the region where the velocities of propagation of heat perturbations and longitudinal elastic vibrations are close in value (b ~ 1), both waves have the same attenuation coefficient $\bar{\delta}_1 = \bar{\delta}_2 = (N - A)/4$. It should be noted that the attenuation coefficients obtained with the help of the asymptotic approximations in [1] and by the method of surfaces of discontinuity in the present paper coincide.

For the Maxwell functions of relaxation

$$\alpha(t) = \frac{\lambda}{\tau_q}\exp\left(-\frac{t}{\tau_q}\right);\ \beta(t) = \frac{c_v}{\tau_e}\exp\left(-\frac{t}{\tau_e}\right);$$
$$\gamma(t) = \frac{\varkappa_2}{\tau_\sigma}\exp\left(-\frac{t}{\tau_\sigma}\right)$$

Eq. (42) assumes the form

$$\delta_{1,2} = \frac{\left(\frac{u_n}{c_1}\right)^2 N + b^2}{2b^2\tau_q}\ \frac{\left(\frac{u_n}{c_1}\right)^2 - 1}{\left(\frac{u_n}{c_1\sqrt{b}}\right)^4 - 1} + \frac{\varepsilon^*\Gamma}{2b^2\tau_\sigma}\ \frac{\left(\frac{u_n}{c_1}\right)^2}{\left(\frac{u_n}{c_1\sqrt{b}}\right)^4 - 1}, \tag{44}$$

where $N = \tau_q/\tau_e$, $\Gamma = \tau_q/\tau_\sigma$.

The graphs of dependences of the coefficients of attenuation of thermoelastic waves on the dimensionless velocity of the thermal perturbations, constructed from Eq. (44), are shown in Fig. 2. For the parameter Γ, we obtain the estimate $\Gamma = 10^{-5}$ ($N = 100$) for the relaxation time of the heat flow $\tau_q = 10^{-11}$ sec [6], the relaxation time of the internal energy $\tau_e = 10^{-13}$ sec [7], and the time of the temperature relaxation of stresses $\tau_0 = 10^{-7}$ sec [8]. The contribution to the attenuation coefficient from the thermal relaxation of the stresses is less than 1% in this case, and we can ignore the second term in (44). Taking account of the thermal memory results in an increase in the attenuation of the thermoelastic waves by a factor of $\left[\frac{N}{b^2}\left(\frac{u_n}{c_1}\right)^2 + 1\right]$ in comparison with the approximation of generalized thermomechanics (for the uncoupled case, by a factor of $1 + N$).

The method of the surfaces of discontinuity, considered in this paper, proved to be effective in studying the propagation of thermoelastic waves in media with thermal memory. The expressions obtained for the velocities and attenuation of the waves may find application in the experimental testing of models of thermoelasticity and the determination of explicit expressions for the relaxation functions of the heat flow and internal energy.

NOTATION

z, the coordinate normal to the surface of the half-space; t, time; T_0, temperature of the nonstressed state of the half-space; $\alpha(t)$, relaxation function of the heat flow; $\beta(t)$, relaxation function of the internal energy; $\gamma(t)$, temperature relaxation function of stresses; $c_V = D_\theta E(\Lambda_0)$, instantaneous volumetric heat capacity; $\varkappa_i$, linearization coefficients; ρ, density; $u_z = u$, displacements normal to the surface of the half-space; $\sigma_{zz} = \sigma$, normal stresses; T, temperature; $\varepsilon = u_{,z}$, deformation; $g = \partial T/\partial z$, temperature gradient.

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